Math CAMPPP 2011

Plenary 1

What's the Focus?

An Introduction to Algebraic Reasoning

Ruth Beatty and Cathy Bruce

Who Are We?

- Ruth: Researcher and Professor at Lakehead University; Recognized expert on young students' algebraic thinking and clinical interviews as a method of learning about student thinking
- Cathy: Researcher and Professor at Trent University; 14 years as a classroom and resource teacher; Recognized expert on models of teacher professional learning in mathematics

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What is our Focus?

- We will focus our discussions on Mathematics: Specifically Linear Relationships
- This area it continues to be difficult for students and is foundational in algebraic reasoning
- We will explore three important constructs in detail:

I. **Multiplicative Thinking**
II. **Generalizing**
III. **Multiple Representations**

Across the Grades

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Layered Learning

- We will look to the research to help us understand how students develop algebraic thinking
- We also challenge you to think about how this connects and applies to the grades you teach/work with
- We will practice responding to student work (in the moment and over time) using student works samples and video

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Guess My Rule

Input	Output
6	69
4	47
0	3
10	113

My rule is:

Output = Input x 11 + 3

Additive and Multiplicative Thinking

- Patterning, even at a basic level of skip counting, can play an important role in the *development* of multiplicative reasoning
- Early patterning experiences of young children often involve simple repetition using one variable (e.g., blue, red, blue, red) which may account for difficulties that older students face in recognizing and generalizing patterns and relationships.
- Often, the teaching of patterning focuses on additive thinking, rather than on multiplicative thinking (necessary for understanding functional relationships)

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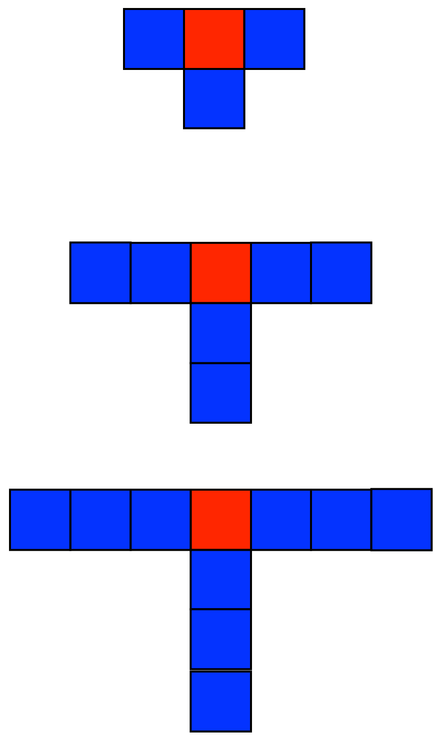
What is Additive Thinking?

When students use additive thinking, they consider the change in only one set of data.

1

2

3



Input	Output
1	4
2	7
3	10
4	13
5	16

+3

+3

+3

+3

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Limitations of Additive Thinking

- Additive thinking allows student to describe the pattern (e.g., add three blue tiles each time) and extend the pattern sequentially
- It does not allow for the prediction of terms far down the sequence, for instance the 100th or 375th position (or term)
- It does not allow for finding the mathematical structure of a pattern, and articulating this as a pattern rule (or equation)

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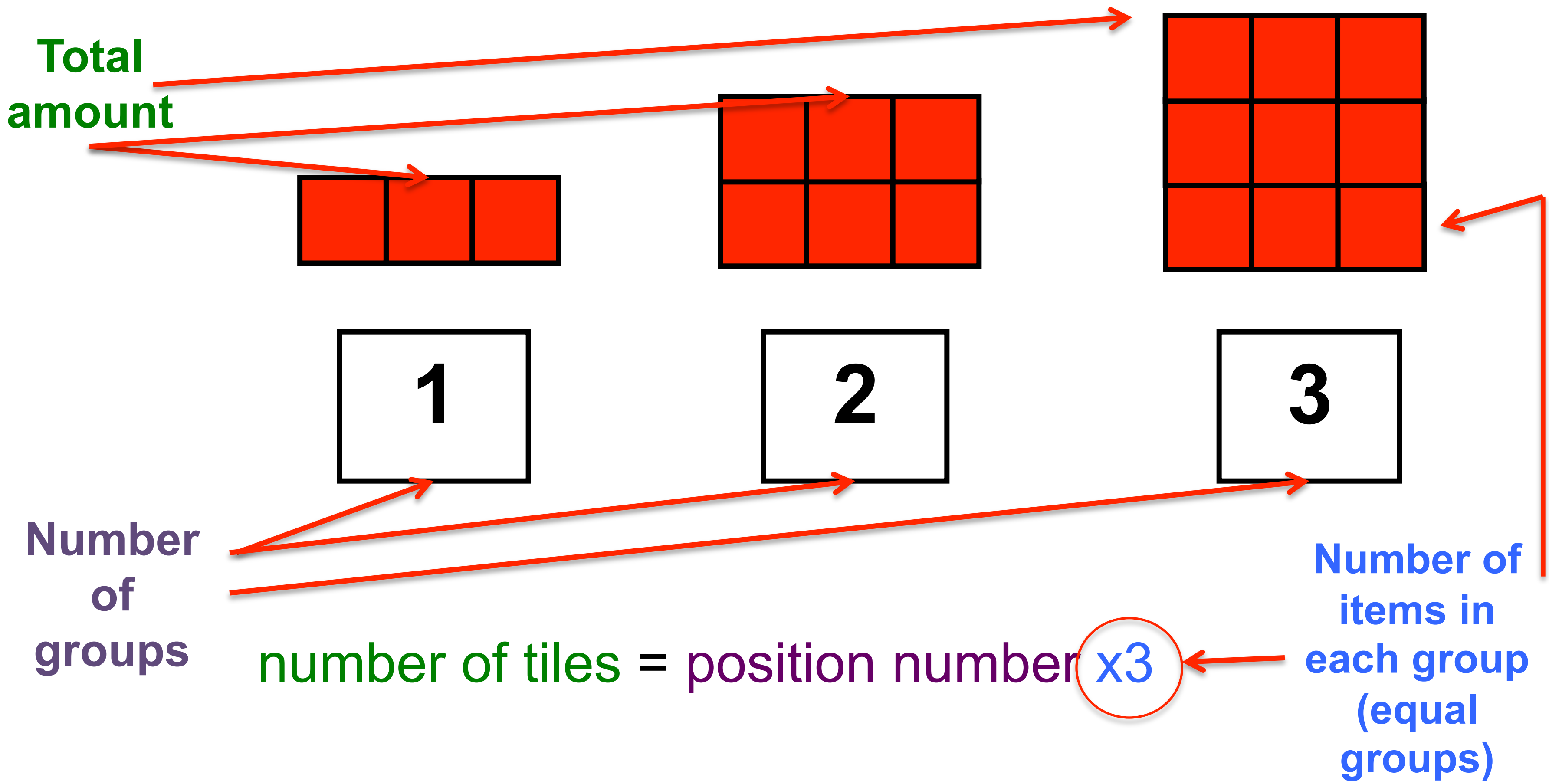
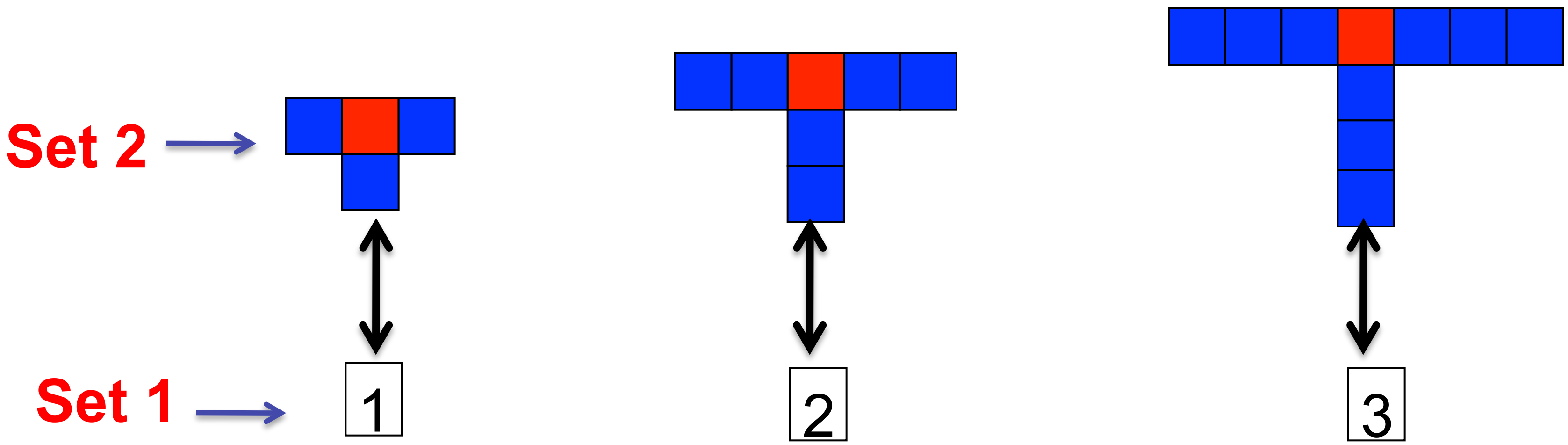
What is Multiplicative Thinking?

Multiplicative situations involve 3 concurrent ideas: **number of items in a group**, **number of groups**, and **a total amount**



What is Multiplicative Thinking?

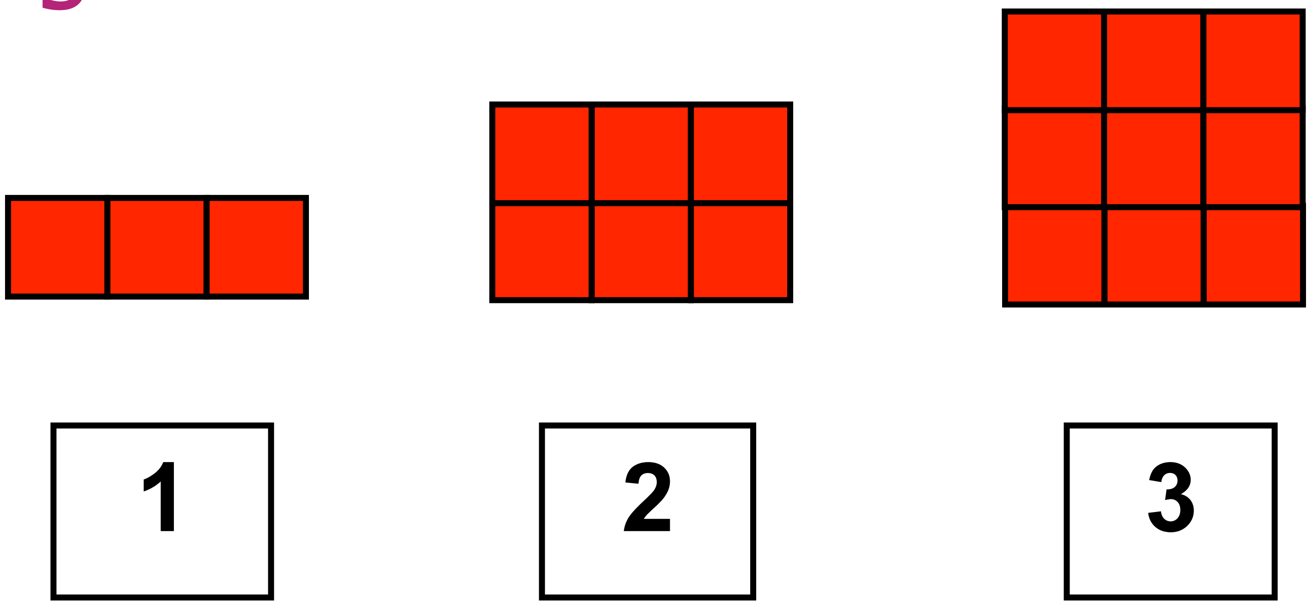
In Algebra, multiplicative thinking also involves understanding the *co-variation* of two sets of data [set 1 (term number or x value), set 2 (total amount of tiles or y value), and the rule between set 1 and set 2]



The mathematical structure of a pattern rule: # of tiles = position number x3+1

This allows students to confidently predict the number of tiles for any term of the pattern.

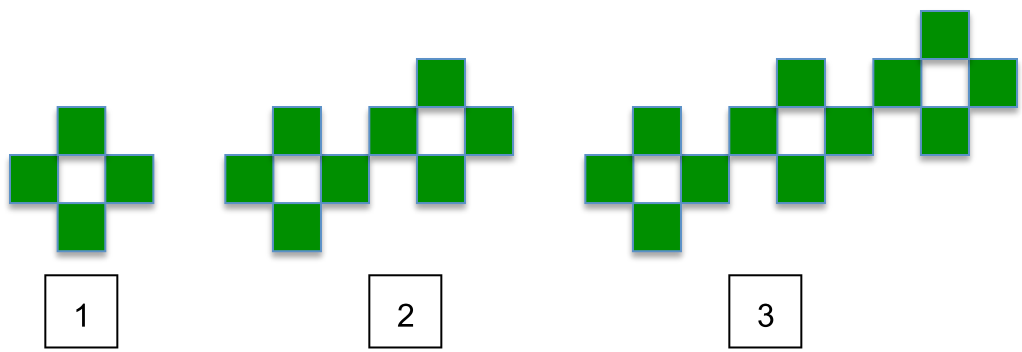
Multiplicative Thinking and Pattern Building



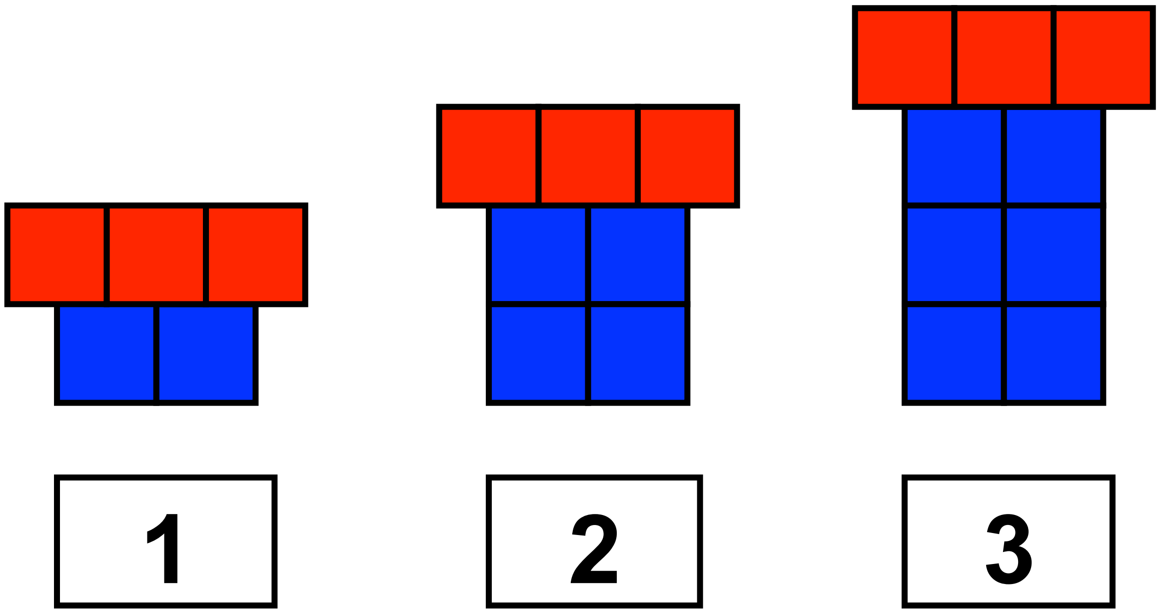
Total number of tiles = position number x3

Unitizing and Pattern Building

- Students develop a concept of “unitizing” – for instance, in the pattern below four green tiles arranged in a checkerboard configuration become the “unit” or “core” of the pattern – the four tiles are simultaneously “four” tiles and “one” pattern core
- The number of iterations of the unit depends on the position number (term number)
- This can also support a developing understanding of the relationship between an independent variable (position number) and a dependent variable (tiles)

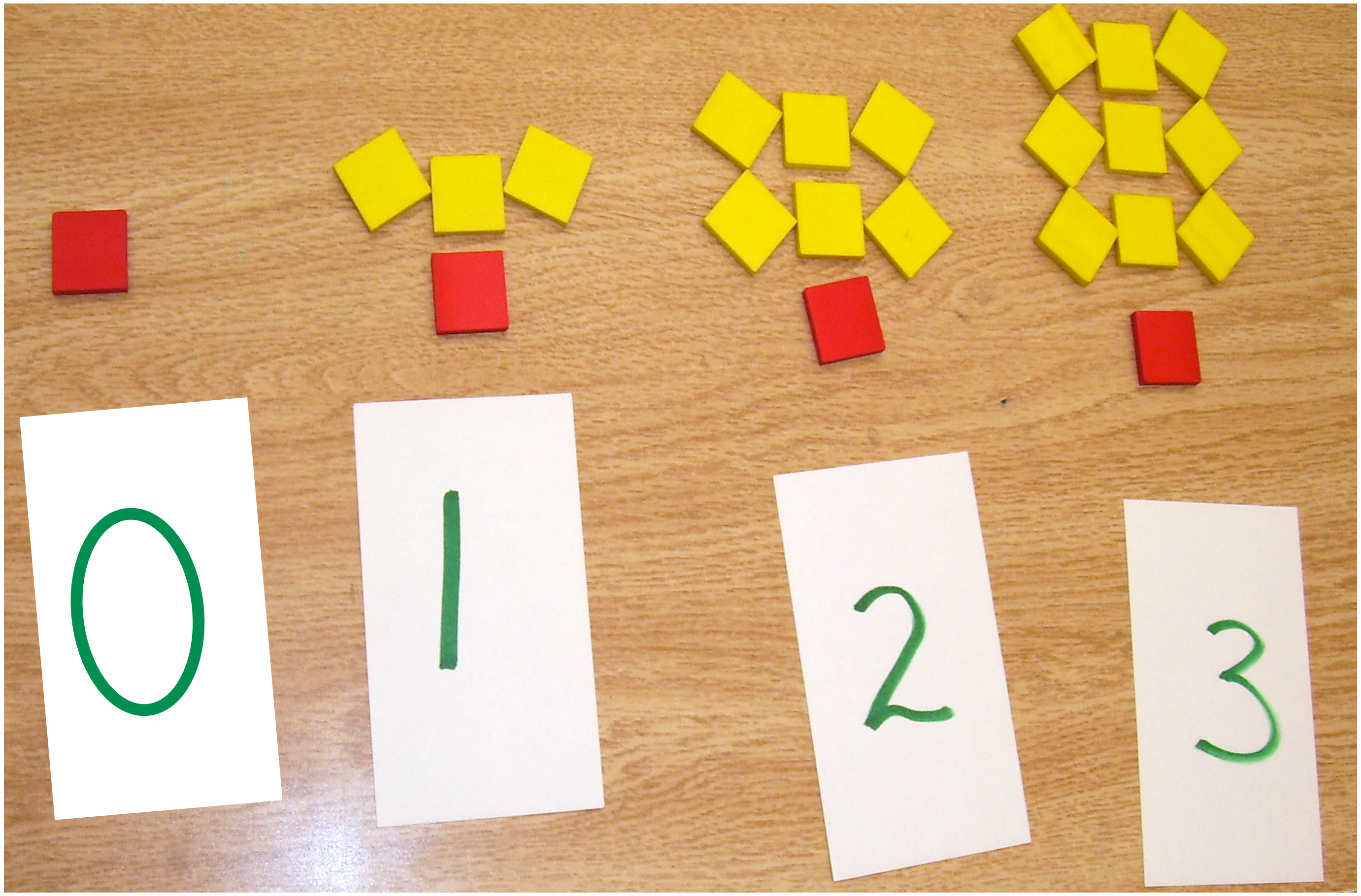


Pattern Building: A Two-Step Rule



Total number of tiles = position number x2+3

What's The Rule?



Your Turn!

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Generalizing

- Patterns offer an initial way for students to grapple with the notion of generalizing
- Algebra can be thought of as generalizations of laws about relationships between and among numbers and patterns
- One form of generalizing is making "far predictions"
- Another form of generalizing is being able to find and articulate a "pattern rule" or equation that represents the mathematical structure of a pattern

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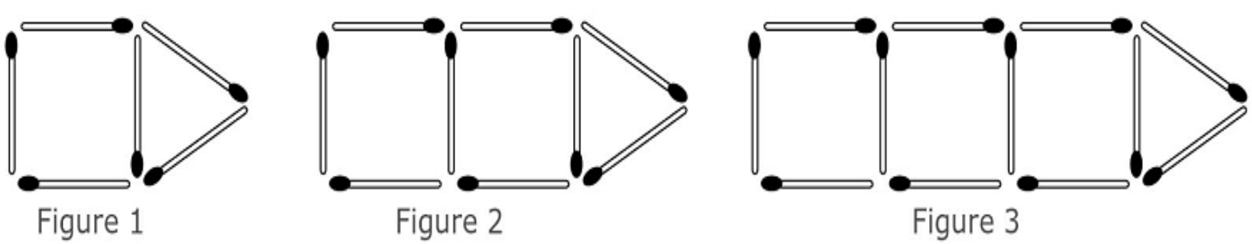
Some Student Difficulties

- A difficulty for many students is understanding that the "pattern rule" or equation must hold for *any* iteration of the pattern
- Students often consider only one term of the pattern when generating a rule
- Once they select a rule, students tend to persist that their rule is correct *even when finding a counter example* (they tend to refute the example, rather than their rule)
- For example, most students consider the rule below to be **$4x+2$**



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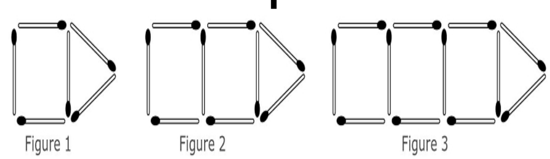
Why Is Generalizing Important?

- Allows students to develop rigour in their mathematics thinking and a commitment to providing justifications
- Encourages "What IF" thinking
- Helps students make near and far predictions
- Helps students develop powerful reasoning

For example:



Actions:
Physical manipulations
Mental configurations

Prediction:
Number of matchsticks needed for 10^{th} term or 20^{th} term

Generalized Rule:
Recognizing the relationship between the figure number and number of matchsticks

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Multiple Representations

- We have already used multiple representations of linear growing patterns
- Research indicates that working with visual representations and deconstructing these in order to identify the relationship between variables is a *more successful* method of developing generalized algebraic formulae than either working with number sequences (using ordered tables of values) or memorizing rules for transforming equations

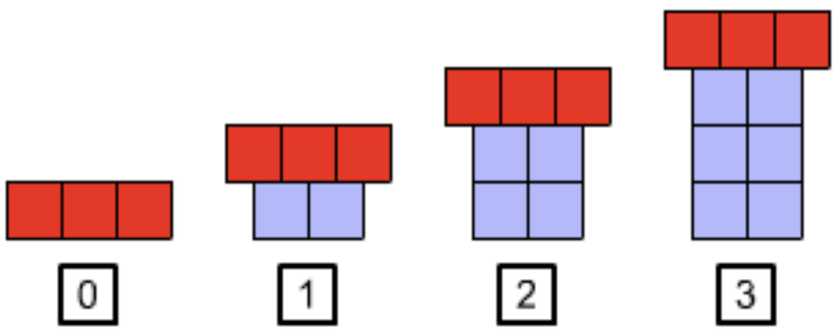
Multiple Representations

- Representations fall along a continuum from concrete to symbolic
  - Concrete (e.g., building linear growing patterns with tiles)
  - Semi-Abstract/Abstract (visual representations of patterns, diagrams)
  - Graphical (graphs)
  - Symbolic (pattern rules, equations)

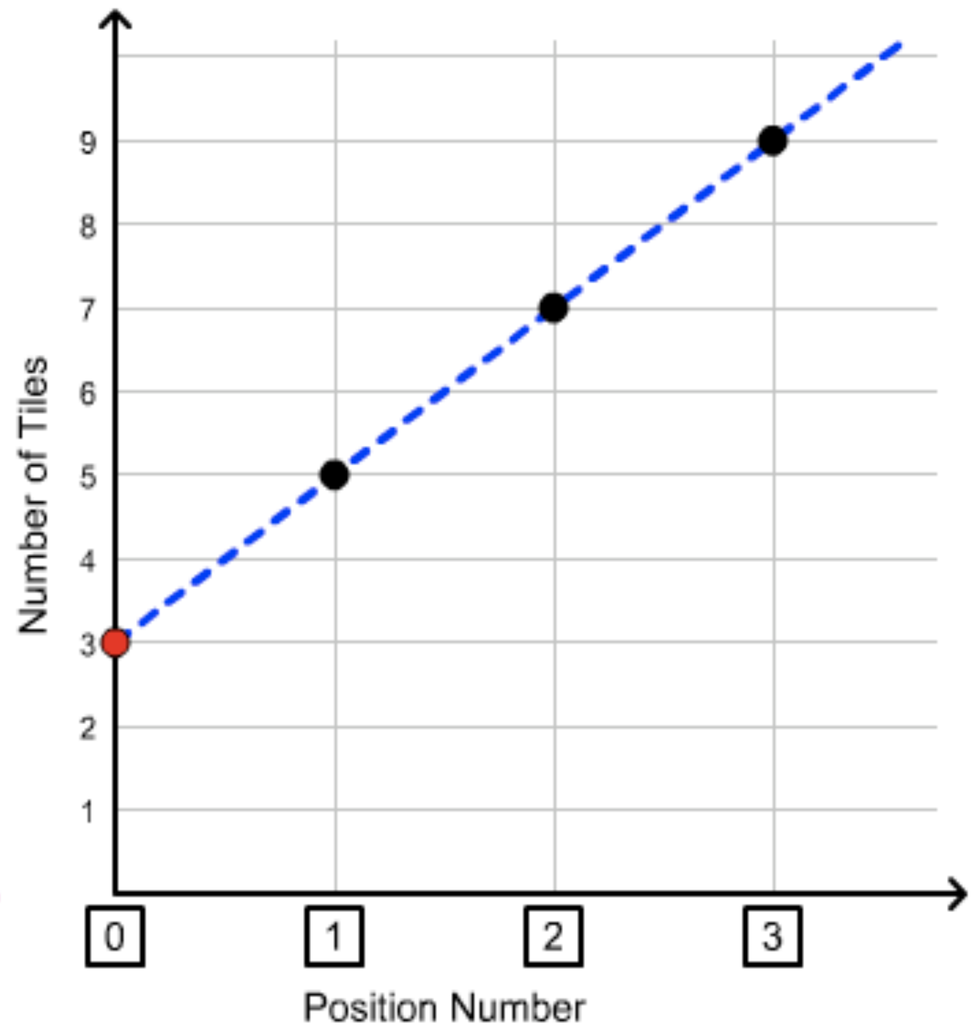
Multiple Representations

- Math education researchers recommend that students be introduced to a variety of representations (concrete, abstract, graphical, symbolic) in order to develop a deep conception of algebraic relationships
- Students require opportunities to explore the *interaction* between relationships in order to make connections, and predict how changes in one representation result in transformations of other representations

Connecting Patterns and Graphs

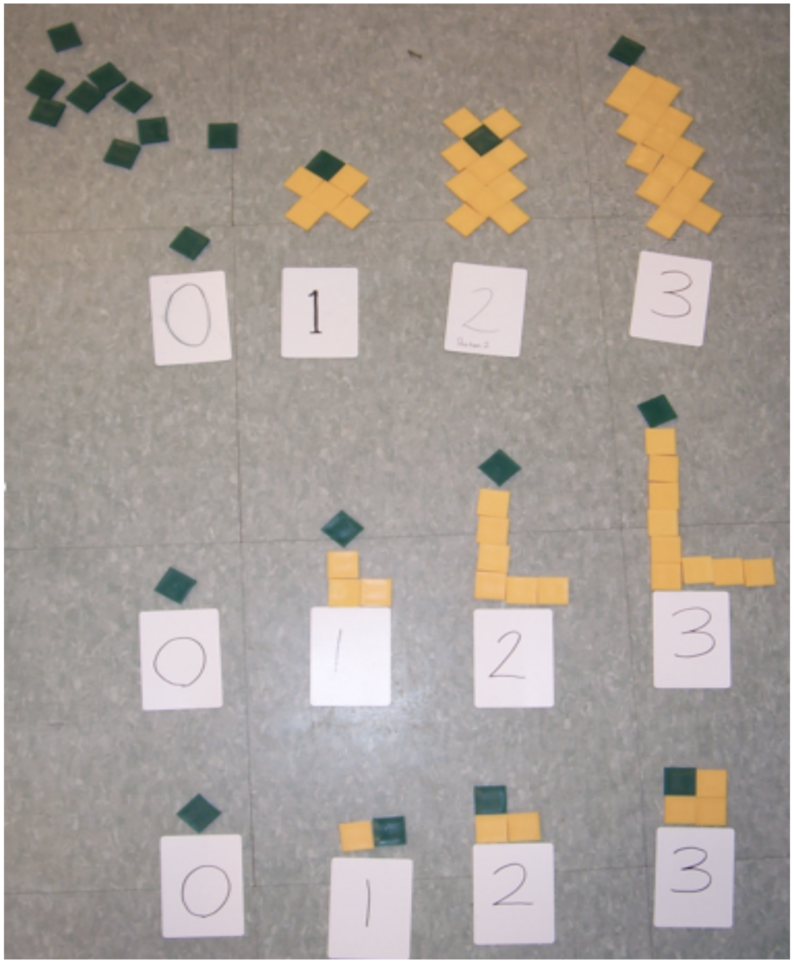


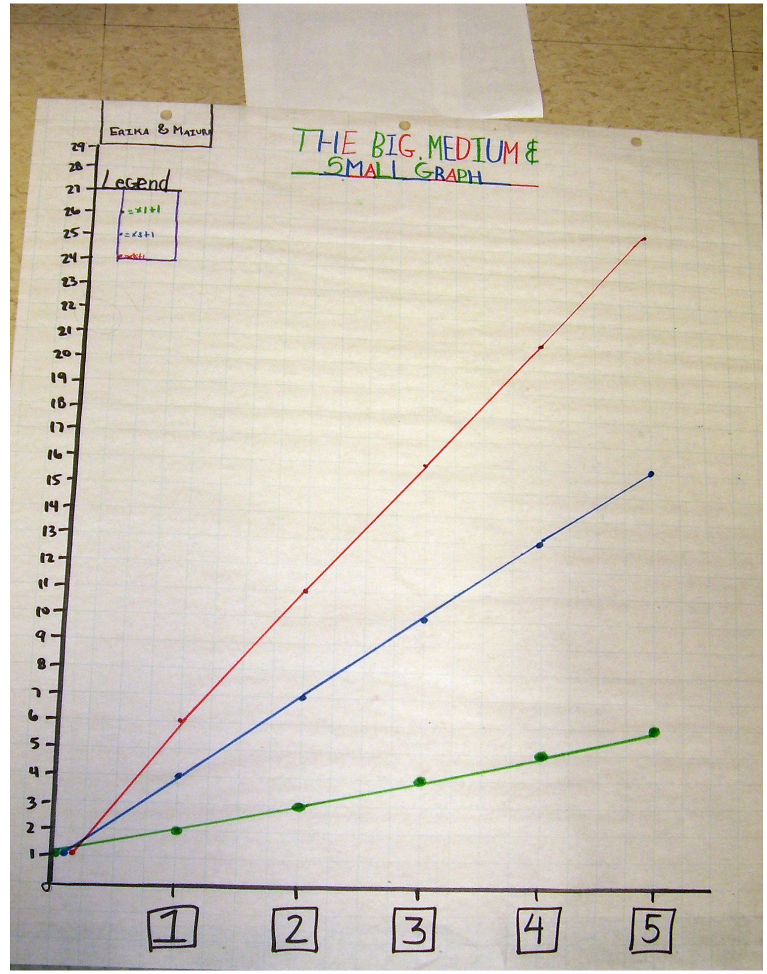
Number of Tiles = (Position Number) x 2 + 3



Multiple Representations

- Students can explore the connections among the representations, for example, how changing the value of the multiplier or the constant in the pattern rule affects
  - The linear growing pattern (how will the tiles change if we….?)
  - The graphical representation (how will the graph change if we…?)
- Students are also encouraged to find similarities and differences within and among the different sets of rules…





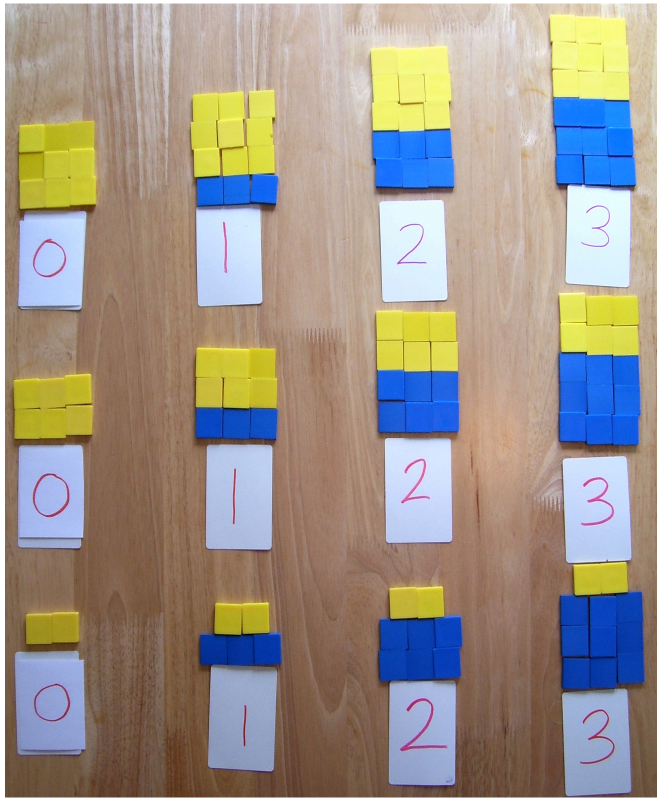
Pattern Rules

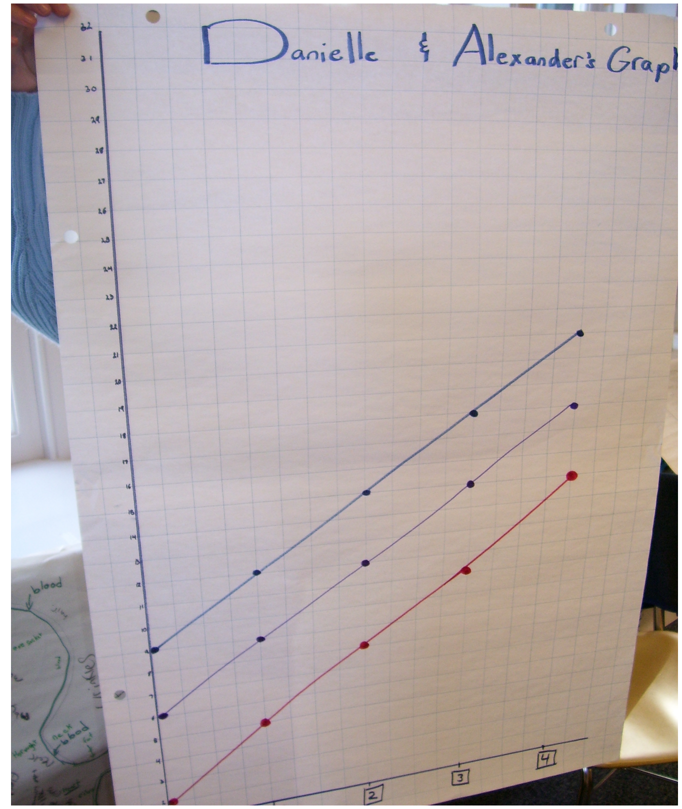
Tiles = position number x1+1

Tiles = position number x3+1

Tiles = position number x5+1

What is similar in the 3 rules? What is different?
What is similar in the 3 patterns? What is different?
What is similar about the trend lines on the graph? What is different?





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Tiles = position number x3+2
Tiles = position number x3+6
Tiles = position number x3+9

Multiple Representations

- The multiplier is responsible for the slope of the trend line
- The constant tells you "where the line starts" [the *y*-intercept]
- A higher multiplier results in a steeper trend line
- Rules that have the same multiplier result in parallel trend lines – there is no *x*-value (position number) at which these lines would intersect (sets with no solution)

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Story Problems

Context or story problems offer another way for students to think about the relationships between quantities.

BUTTERFLY HOUSE…

Some children visited the Butterfly House at the Zoo.
They learnt that a butterfly is made up of 4 wings, one body and two feelers. While they were there, they made models and answered some questions.

For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

_______________ wings

_______________ bodies

_______________ feelers

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

c. How many wings, bodies and feelers will be needed to make 98 model butterflies. **Show all your working and explain your answer in as much detail as possible.**

_______________ wings

_______________ bodies

_______________ feelers

d. How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers? **Show all your working and explain your answer in as much detail as possible.**

e. To feed 2 butterflies the zoo needs 5 drops of nectar per day. How many drops would they need each day for 12 butterflies? **Show all your working and explain your answer in as much detail as possible.**

iMusic Purchase Plans Problem

Thursika and Dave both loved downloading music. They found a great internet site that had two different payment schemes for downloading music.

Plan A was to pay a membership fee of $16 before downloading any music, and pay $2 for each album.

Plan B was to pay $5 for each album, but you only had to pay $1 as a membership fee before downloading music.

Thursika chose **Plan A**.
Dave chose **Plan B**.

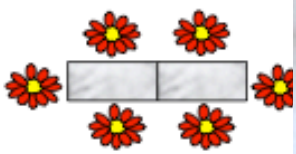
1. If they both downloaded 10 albums, who made the better choice of payment plan?

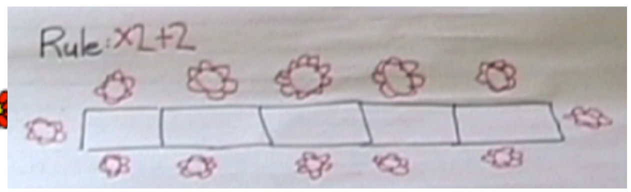
Chapter One – The First Flower Path Rule

Grandma Splendido loved her garden very much. She loved it so much that she decided to design a beautiful path to meander through the garden.

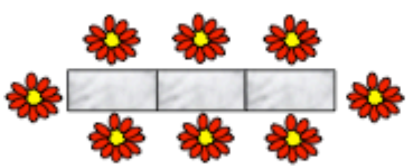
She built a one stone flower path that looked like this:

She decided that a longer path would be better, and so she added a stone. Her path looked like this:





Then she decided that she needed an even longer path that looked like this:



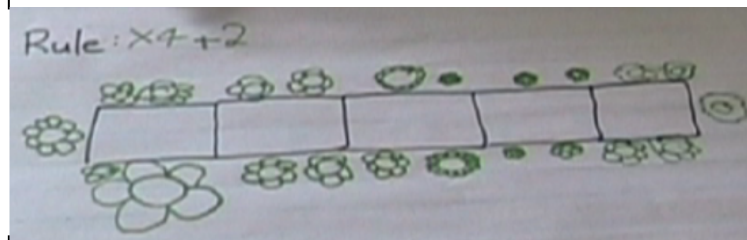
Grandma Splendido was **pooped**. She was NOT happy with her flower path, but was too too tired to build any more flower paths!

Ferdie Splendido, her grandson, came to the rescue.

"If we can figure out a rule for your flower path, we can **draw a diagram** of what it will look like, and we can **graph** it so we know exactly how many flowers to buy!!"

"I don't like this sort of path," said Grandma. She looked at the rule, and changed it so it followed the rule:

total number of flowers = paving stones **x 4 +2**.



"BLECK", said Grandma Splendido. "Let's try again with a new rule! Change it to:

total number of flowers = paving stones **x 2+6**!"